



# Simplifying Heuristic Search and Domain Modeling With F#



### Talk Outline

- Introduction
  - Grid Navigation Problem Setup
  - Example Domains Toys
  - Example Domains Industrial
- An Algorithm: A\*
- Fitting Domains To Interface
- Conclusion



- Search is a technique for solving problems
- These problems look like this:
  - States
  - Actions
  - Goals
  - Heuristics

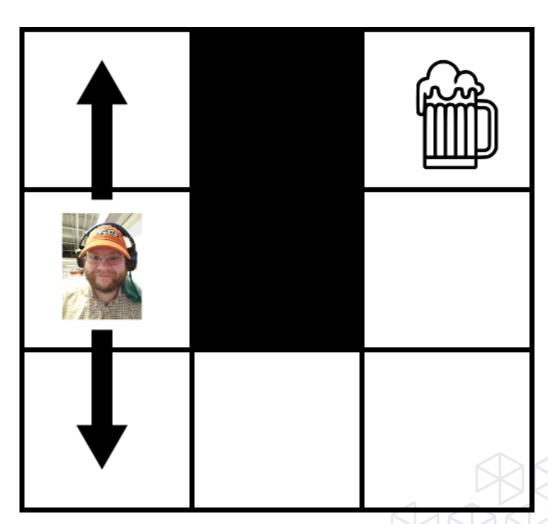


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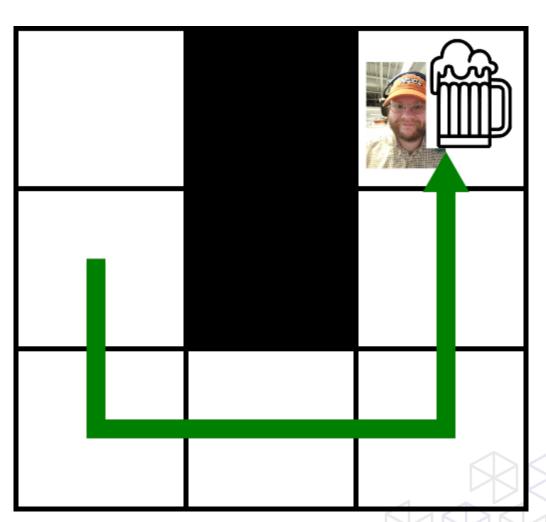


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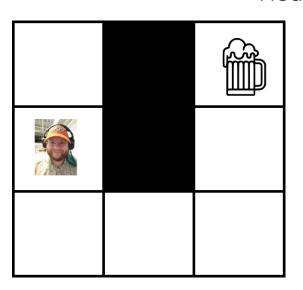


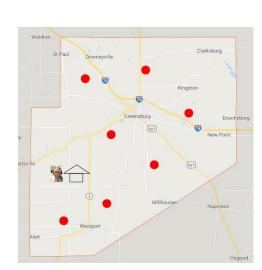
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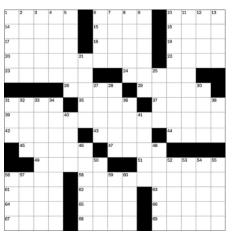




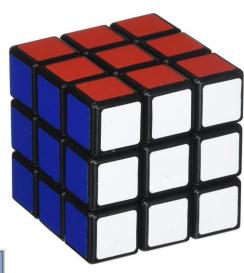
- Search is a *general* technique for solving problems
- The search cares about your problem using this abstraction:
  - States
  - Actions
  - Goals
  - Heuristics





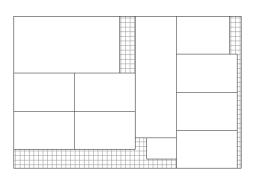


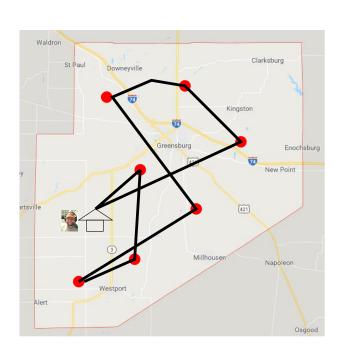


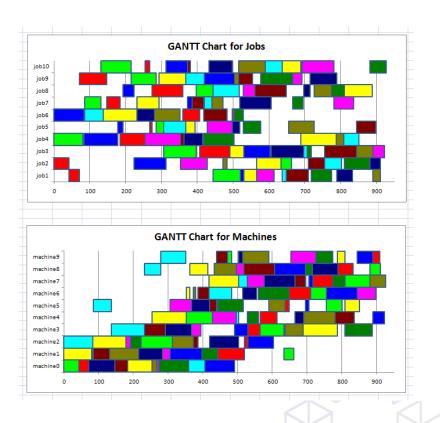




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- Search is a technique for solving problems
- These problems look like this:
  - Graphs
    - Nodes
    - Edges
  - Functions on Nodes
    - Goal predicate
    - Heuristic Estimate

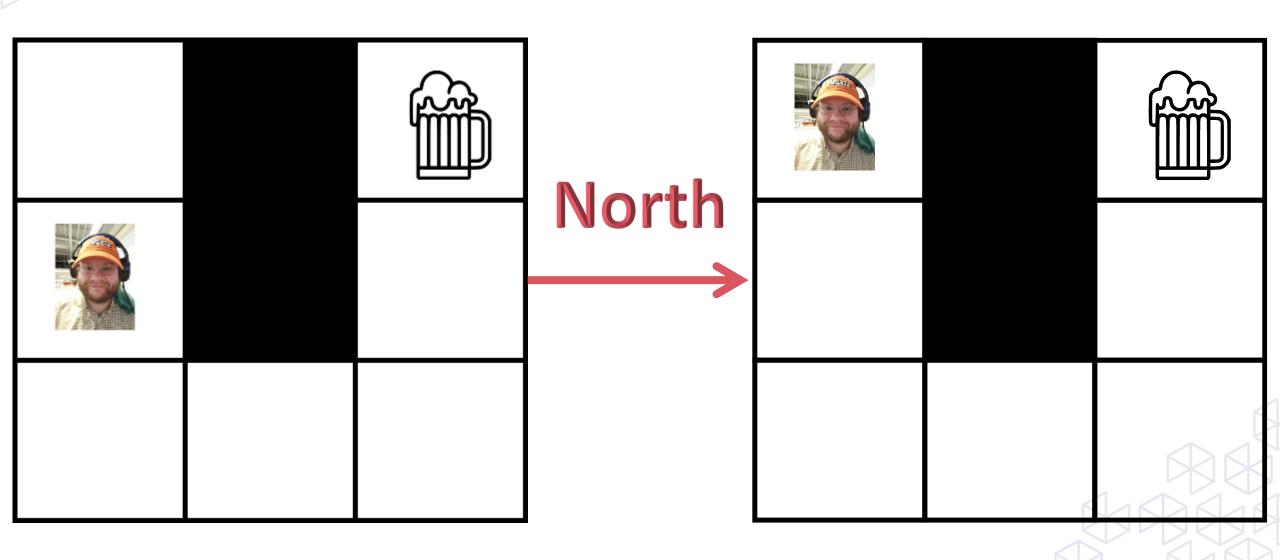


```
type Edge<'state, 'action when 'state: comparison and 'action: comparison > =

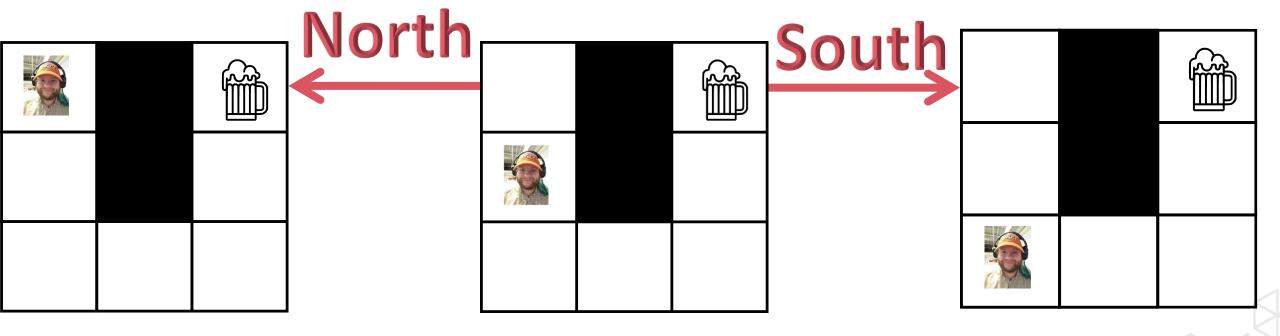
{
    origin : 'state
    dest : 'state
    by_way_of : 'action
}

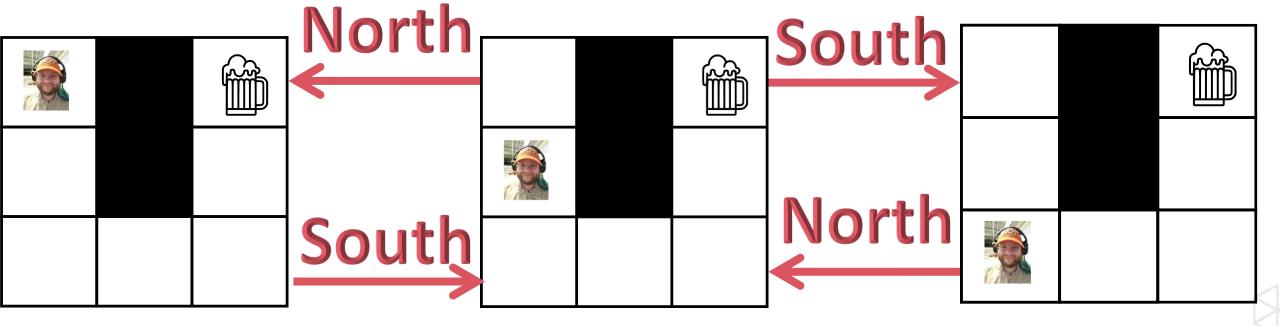
type Graph<'state, 'action when 'state: comparison and 'action: comparison> = {
    nodes : Set<'state>;
    edges : Set<Edge<'state, 'action>>;
}
```









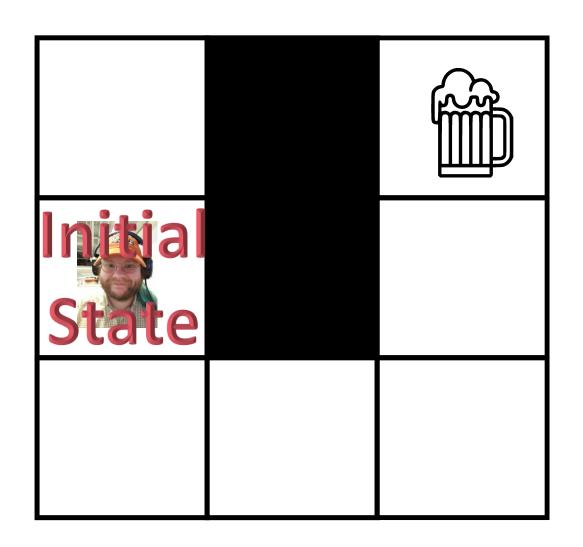


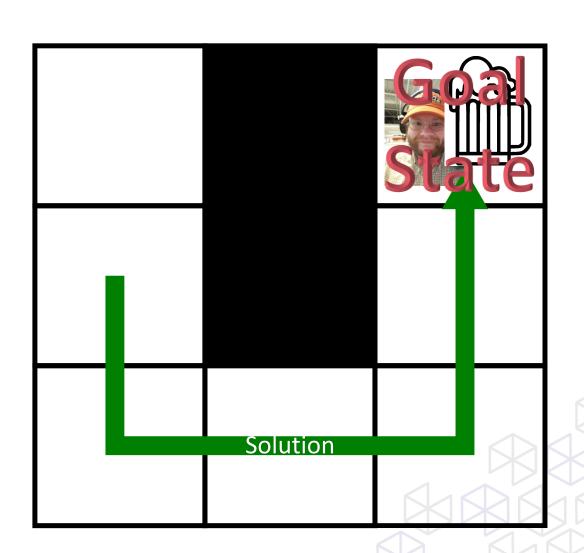
```
type Edge<'state, 'action when 'state: comparison and 'action: comparison > =

{
    origin : 'state
    dest : 'state
    by_way_of : 'action
}

type Graph<'state, 'action when 'state: comparison and 'action: comparison> = {
    nodes : Set<'state>;
    edges : Set<Edge<'state, 'action>>;
}
```







```
type GraphSearchProblem<'state, 'action when 'state: comparison and 'action: comparison > = {
    graph : Graph<'state, 'action>;
    initial_state : 'state;
    goals : Set<'state>;
}

type Solution<'state, 'action when 'state: comparison and 'action: comparison> = {
    initial_state : 'state;
    path : Edge<'state, 'action> list;
}
```



Explicitly representing the graph is often infeasible due to size!

```
type GraphSearchProblem<'state, 'action when 'state. comparison and 'action: comparison > = {
    graph : Graph<'state, 'action>,
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}

type Solution<'state, 'action when 'state: comparison and 'action: comparison> = {
    initial_state : 'state;
    path : Edge<'state, 'action> list;
}
```

```
type ImplicitGraphSearchProblem<'state, 'action when 'state: comparison and 'action: comparison > = {
    initial_state : 'state
    goal_p : 'state -> bool
    expand : 'state -> Set<Edge<'state, 'action>>
}

// An ImplicitGraphSearchProblem can be converted into a GraphSearchProblem by
// taking the transitive closure of initial_state and expand
```



```
type GraphSearchProblem<'state, 'action when 'state: comparison and 'action: comparison > = {
    graph : Graph<'state, 'action>;
    initial_state : 'state;
    goals : Set<'state>;
}

type Solution<'state, 'action when 'state: comparison and 'action: comparison> = {
    initial_state : 'state;
    path : Edge<'state, 'action> list;
}
```

So don't represent the graph explicitly. Generate it lazily.

```
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// An ImplicitGraphSearchProblem can be converted into a GraphSearchProblem by
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```
type HeuristicSearchProblem<'state, 'action when 'state: comparison and 'action: comparison > = {
    initial_state : 'state
    goal_p : 'state -> bool
    expand : 'state -> Set<Edge<'state, 'action>>
    heuristic : 'state -> float
}

type HeuristicSearchAlgorithm<'state, 'action when 'state: comparison and 'action: comparison> =
    ('state -> Set<Edge<'state, 'action>) -> ('state -> bool) -> ('state -> float) -> 'state -> Edge<'state, 'action> list
```



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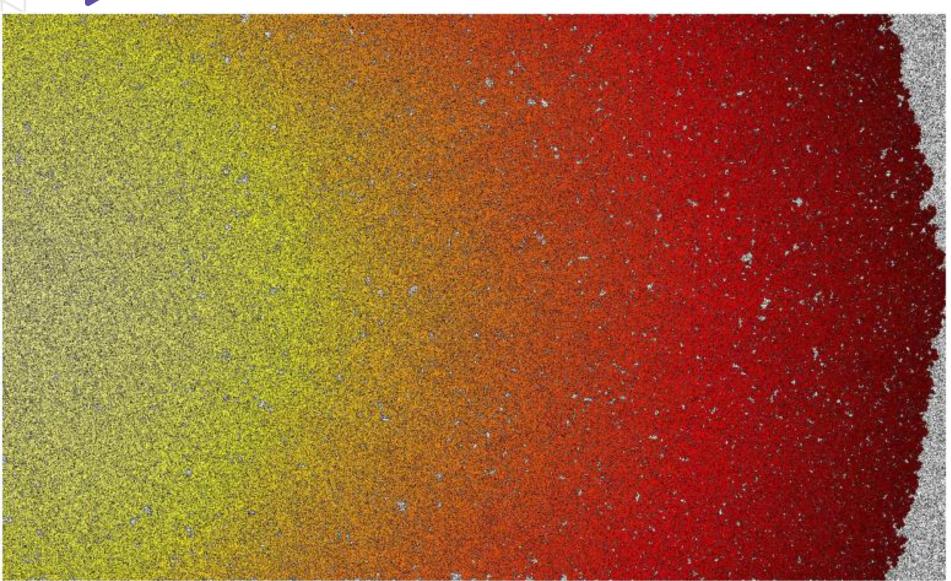
# > Heuristic Search

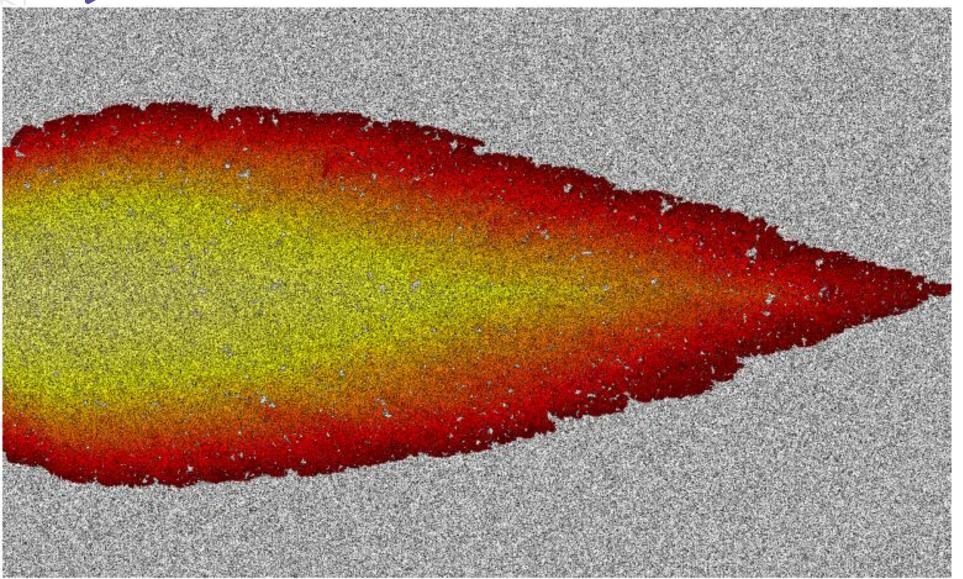






# > Heuristic Search





```
52
     type HeuristicSearchProblem<'state, 'action when 'state: comparison and 'action: comparison > = {
         initial state : 'state
         goal p : 'state -> bool
54
         expand : 'state -> Set<Edge<'state, 'action>>
55
         heuristic : 'state -> float
57
     type HeuristicSearchAlgorithm<'state, 'action when 'state: comparison and 'action: comparison> =
          ('state -> Set<Edge<'state, 'action>>) -> ('state -> bool) -> ('state -> float) -> 'state -> Edge<'state, 'action> list
                                                                                                            Solution
                        Expand
                                                       Goal
                                                                            Heuristic
                                                                                              Initial
```



### Graph search

```
function GRAPH-SEARCH (problem, fringe) returns a solution, or failure
   closed \leftarrow an empty set
   fringe \leftarrow Insert(Make-Node(Initial-State[problem]), fringe)
   loop do
       if fringe is empty then return failure
       node \leftarrow Remove-Front(fringe)
       if GOAL-TEST(problem, STATE[node]) then return node
       if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe \leftarrow InsertAll(Expand(node, problem), fringe)
   end
```

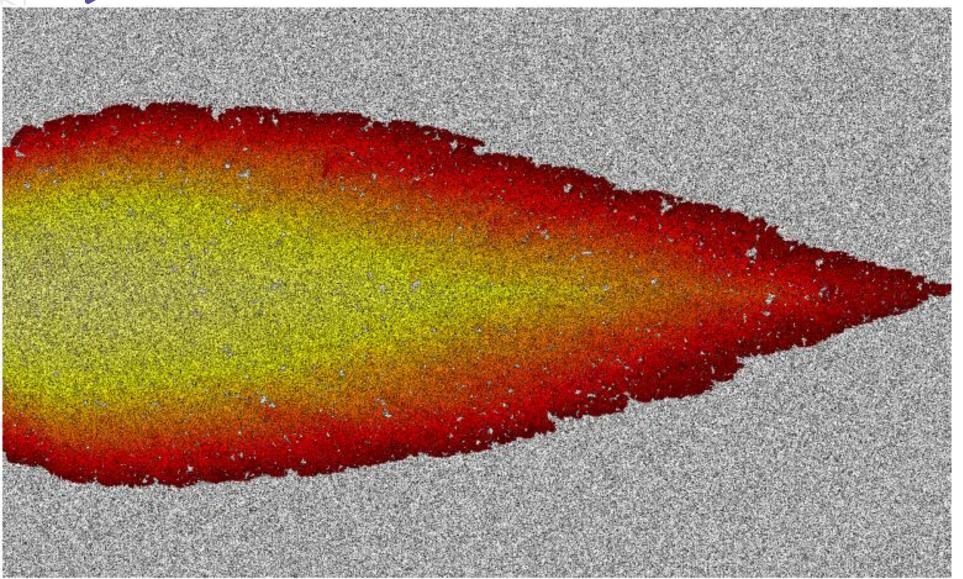


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   end
```



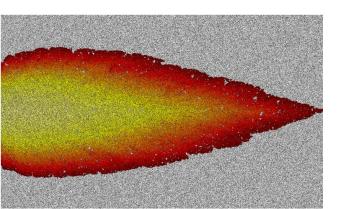
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$$f(n) = g(n) + h(n)$$

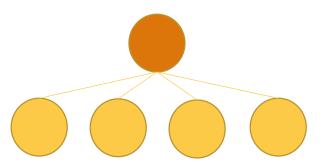


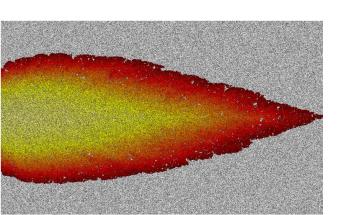
And so on and so forth...



$$f(n) = g(n) + h(n)$$



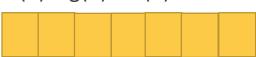


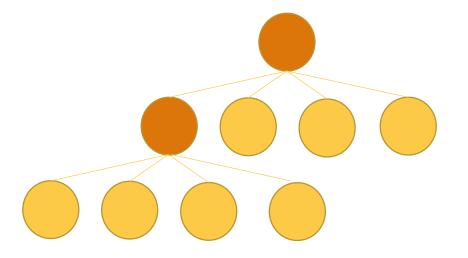


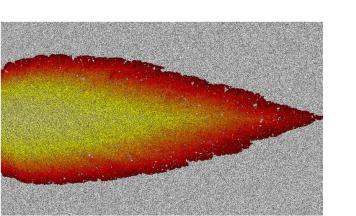
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$$f(n) = g(n) + h(n)$$





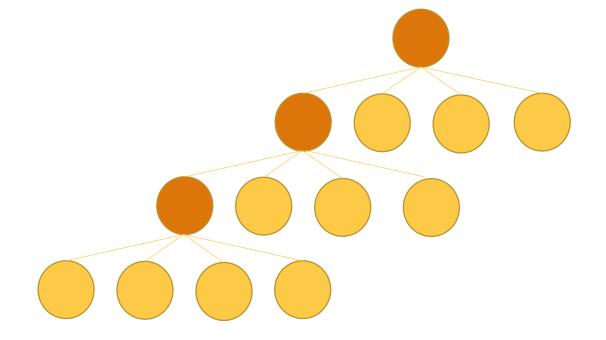


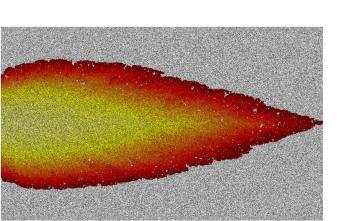
And so on and so forth...



### > Heuristic Search – A\*

$$f(n) = g(n) + h(n)$$



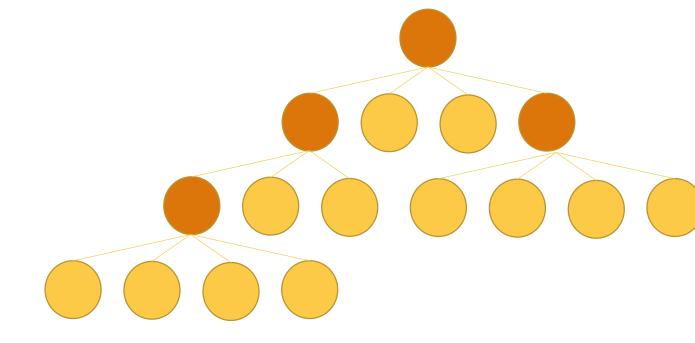


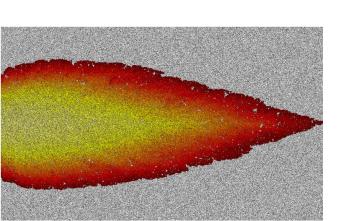
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### Heuristic Search – A\*

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       if Goal-Test (problem, State[node]) then return node
       if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe \leftarrow InsertAll Expand (node, problem), fringe)
   end
```

98

### **Heuristic Search – A\***

type HeuristicSearchAlgorithm<'state, 'action when 'state: comparison and 'action: comparison> =

metrics with stop time = Some DateTime.Now }

```
('state -> Set<Edge<'state, 'action>>) -> ('state -> bool) -> ('state -> float) -> 'state -> Edge<'state, 'action> list
                                                                                                         Solution
                       Expand
                                                      Goal
                                                                          Heuristic
                                                                                           Initial
82
           while not finished do
               match pop() with
84
                 None -> finished <- true
                 Some current node ->
                   if node goal test current node then begin
                       metrics.solution nodes <- { solution = current node; found at time = DateTime.Now } :: metrics.solution nodes;
87
                       finished <- true end else
                       begin
                           let key val = node key current node
90
                           let should expand = consider node key val current node
91
                           if should expand then begin
                               closedlist <- closedlist Add(key val, current node)
93
                               let child tuples = node expand current node
                               List.iter (consider child current node) child tuples
95
96
                            end
                       end
```

### **Heuristic Search – A\***

```
type HeuristicSearchAlgorithm<'state, 'action when 'state: comparison and 'action: comparison> =

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Expand

Goal

Heuristic

Initial

Solution

While not finished do

match pop() with

None -> finished <- true
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```
None -> finished <- true
                 Some current node ->
                   if node goal test current node then begin
                       metrics.solution nodes <- { solution = current node; found at time = DateTime.Now } :: metrics.solution_nodes;
87
                       finished <- true end else
                       begin
                           let key val = node key current node
90
                           let should expand = consider node key val current node
91
                           if should expand then begin
                               closedlist <- closedlist.Add(key_val, current_node)</pre>
93
                               let child tuples = node expand current node
                               List.iter (consider child current node) child tuples
95
96
                            end
                       end
98
             metrics with stop time = Some DateTime.Now }
```



84

90 91

96

### Heuristic Search – A\*

### Graph search

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure

closed ← an empty set

fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)

loop do

if fringe is empty then return failure

node ← REMOVE-FRONT(fringe)

if GOAL-TEST(problem, STATE[node]) then return node

if STATE[node] is not in closed then

add STATE[node] to closed

fringe ← INSERTALL(EXPAND(node, problem), fringe)

end
```

```
while not finished do
    match pop() with
     None -> finished <- true
     Some current node ->
        if node goal test current node then begin
            metrics.solution nodes <- { solution = current_node; found_at_time = DateTime.Now } :: metrics.solution_nodes;</pre>
            finished <- true end else
            begin
                let key_val = node_key current_node
                let should_expand = consider_node key_val current_node
                if should expand then begin
                    closedlist <- closedlist.Add(key val, current node)</pre>
                    let child tuples = node expand current node
                    List.iter (consider child current node) child tuples
                end
            end
 metrics with stop time = Some DateTime.Now }
```

### **Heuristic Search – A\***

```
type HeuristicSearchAlgorithm<'state, 'action when 'state: comparison and 'action: comparison> =

('state -> Set<Edge<'state, 'action>>) -> ('state -> bool) -> ('state -> float) -> 'state -> Edge<'state, 'action> list

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93
                               let child tuples = node expand current node
                               List.iter (consider child current node) child tuples
95
96
                            end
                       end
98
             metrics with stop time = Some DateTime.Now }
```

### **Heuristic Search – A\***

```
□let astar search (iface : DomainInterface.CostHeuristicDuplicateDomainInterface<float, 'state, 'hashvalue>) =
     let mutable openlist : FSharpx.Collections.IPriorityQueue<'state SearchNode> = FSharpx.Collections.PriorityQueue.empty false
     let mutable closedlist : FSharpx.Collections.PersistentHashMap<'hash value, 'state SearchNode> = FSharpx.Collections.PersistentHashMap.empty
     let root = make root iface.H iface.InitialState
     let metrics = initial_metrics ()
     let node key = wrap state fn iface.Key
     let node expand = wrap state fn iface.Expand
     let node_goal_test = wrap_state_fn iface.GoalP
     let enqueue (node : 'state SearchNode) = openlist <- FSharpx.Collections.PriorityQueue.insert node openlist</pre>
     let consider child current node (state, step cost) =...
     let pop () = ...
     let consider node key val current node = ...
     enqueue root
     let mutable finished = false
     while not finished do
         match pop() with
           None -> finished <- true
           Some current node ->
             if node goal test current node then begin
                 metrics.solution nodes <- { solution = current node; found at time = DateTime.Now } :: metrics.solution nodes;</pre>
                 finished <- true end else
                 begin
                     let key val = node key current node
                     let should expand = consider node key_val current_node
                     if should_expand then begin
                         closedlist <- closedlist.Add(key val, current node)</pre>
                         let child tuples = node expand current node
                         List.iter (consider child current node) child tuples
                      end
                 end
       metrics with stop time = Some DateTime.Now }
```



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  - String Edit Distance
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### Heuristic Search – What is a Domain?

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```
■module DomainInterface
     type DomainInterface<'cost, 'state> =
           abstract member Expand : 'state -> ('state * 'cost) list
           abstract member GoalP : 'state -> bool
           abstract member InitialState : 'state
     type DuplicateDomainInterface<'cost, 'state, 'hashvalue> =
8
           inherit DomainInterface<'cost, 'state>
           abstract member Key : 'state -> 'hashvalue
10
           abstract member DisplayKey : 'state -> string
11
```

### Heuristic Search – What is a Domain?

```
    □ module DomainInterface

     \(\begin{align*} \text{type DomainInterface<'cost, 'state> = \)
          abstract member Expand : 'state -> ('state * 'cost) list
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          inherit DomainInterface<'cost, 'state>
          abstract member Key : 'state -> 'hashvalue
10
          abstract member DisplayKey : 'state -> string
11
12
     13
14
          inherit DuplicateDomainInterface<'cost, 'state, 'hashvalue>
          abstract member H : 'state -> 'cost
15
```

```
interface DomainInterface(problem : Problem) =
interface DomainInterface.CostHeuristicDuplicateDomainInterface<float, State, int> with
member this.Expand s = expand problem.board s
member this.GoalP s = goal_test problem s
member this.Key s = key problem.board s
member this.H s = manhattan_distance problem s |> float
member this.InitialState = make_initial_state problem
```

```
type HeuristicSearchAlgorithm<'state, 'action when 'state: comparison and 'action: comparison> =

('state -> Set<Edge<'state, 'action>>) -> ('state -> bool) -> ('state -> float) -> 'state -> Edge<'state, 'action> list
```

```
\(\begin{aligned}
\begin{aligned}
\begin{
18
                                                                                                                                                                     x : int
 19
                                                                                                                                                                  y: int
  20
  21
 22
                                                                                                          type Board = bool [,]
 23
 24
                                                                                       25
                                                                                                                                                                     start : Position
 26
                                                                                                                                                                     finish : Position
  27
                                                                                                                                                                      board : Board
  28
  29
```



```
\(\begin{aligned}
\begin{aligned}
\begin{
18
                                                                                                                                                                      x : int
 19
                                                                                                                                                                   y: int
 20
 21
22
                                                                                                            type Board = bool [,]
23
24
                                                                                       itype Problem = {
25
                                                                                                                                                                       start : Position
26
                                                                                                                                                                      finish : Position
27
                                                                                                                                                                       board : Board
 28
 29
```

```
⊟type Action =
        North
         South
         East
         West
         Noop
     ilet string of action = function
         North -> "North"
         South -> "South"
         East -> "East"
13
         West -> "West"
         Noop ->
```

145

```
⊨let opposite action = function
          North -> South
          South -> North
147
          East -> West
         West -> East
          Noop -> Noop
      idet are opposite (act1 : Action) (act2 : Action) =
            act1 = opposite action act2
      ☐let move (position : Position) = function
          North -> { position with y = position.y - 1 }
         South -> { position with y = position.y + 1 }
         East -> { position with x = position.x + 1 }
         West -> { position with x = position.x - 1 }
         Noop -> position
      ⊟let move state (state : State) action =
             position = move state.position action; generated by = action }
      ⊟let expand (board : Board) (state : State) =
            let possible actions = [North; South; East; West]
            let validate position = legal position board
            let consider action (accum : (State * float) list) (action : Action) =
                if are opposite action state.generated by then accum else
                let state' = move state state action
170
                if validate_position state'.position then
                        (state', 1.) :: accum
                        accum
            List.fold consider_action [] possible_actions
```

```
key (board : Board) (state : State) =
            let width = board.GetLength 1
178
179
            state.position.x + state.position.y * width
180
       □let make_initial_state (problem : Problem) = {
181
            position = problem.start;
182
            generated by = Noop
183
184
185
186
       ilet goal_test (problem : Problem) (state : State) =
            problem.finish = state.position
187
188
       i=let manhattan_distance (problem : Problem) (state : State) =
189
            abs(problem.finish.x - state.position.x) + abs(problem.finish.y - state.position.y)
190
191
```



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52

54

55

56 57 58

```
362
       type StringDomainInterface(problem : Problem) =
            interface DomainInterface.CostHeuristicDuplicateDomainInterface<float, State, string> with
363
                member this.Expand s = expand problem s
364
                member this.GoalP s = goal_test problem s
365
                member this.Key s = key s
366
                member this.DisplayKey s = string of element array s.state
367
                member this.H s = character delta problem s
368
                member this.InitialState = make initial state problem
369
```

```
type HeuristicSearchProblem<'state, 'action when 'state: comparison and 'action: comparison > = {
    initial_state : 'state
    goal_p : 'state -> bool
    expand : 'state -> Set<Edge<'state, 'action>>
    heuristic : 'state -> float
}

type HeuristicSearchAlgorithm<'state, 'action when 'state: comparison and 'action: comparison> =
    ('state -> Set<Edge<'state, 'action>>) -> ('state -> bool) -> ('state -> float) -> 'state -> Edge<'state, 'action> list
```



```
type Element =
        Alphabetical of Character
        Nul1
     type Update = {
          index: int
          character: Character
     42
        Remove of int
43
        Add of Update
        Replace of Update
        ShiftLeft
46
        ShiftRight
47
        Noop
     type State = {
          state: Element array
          generated by : Action
     start: Element array
          finish: Element array
```

```
□let goal test (instance : Problem) (state : State) =
318
           state.state = instance.finish
319
320
      ⊟let key (state : State) =
321
322
           string of element array state.state
323
      324
325
               state = problem.start
326
               generated by = Noop
327
328
329
      ilet character_delta (problem : Problem) (state : State) =
330
331
           let char count = function
             Null -> 0
332
             -> 1
333
           let problem_count = Array.fold (fun accum el -> accum + char_count el) 0 problem.finish
334
           let state count = Array.fold (fun accum el -> accum + char count el) 0 state.state
335
           let delta = problem_count - state_count
336
           if delta < 0 then float delta * ADD COST</pre>
337
338
            else float delta * DEL COST
```

```
□let expand (problem : Problem) (state : State) =
            let possible indices = [0..state.state.Length - 1]
            let possible_characters = appearing_in problem.finish
            let possible updates =
                List.fold (fun accum index ->
                    List.fold (fun accum2 character -> { index = index; character = character } :: accum2) accum possible characters) [] possible indices
            let possible shifts = [ShiftLeft; ShiftRight]
            let possible removes = List.map Remove possible indices
            let possible adds = List.map Add possible updates
307
            let possible replaces = List.map Replace possible updates
            let generate_successor (accum : (State*float) list) (action : Action) =
                if valid move state action then
311
                    (apply nondestructive state action, cost state action) :: accum
312
                else
313
                    accum
            let possible actions = possible shifts @ possible removes @ possible adds @ possible replaces
315
            List.fold generate successor [] possible actions
```



### Talk Outline

- Introduction
- An Algorithm: A\*
- Fitting Domains To Interface
  - Grid Navigation
  - String Edit Distance
- Conclusion



### Talk Outline

- Introduction
- An Algorithm: A\*
- Fitting Domains To Interface
- Conclusion
  - Functional programming is a natural fit for search code
    - Passing in Expand, Goal, Heuristic Functions
    - Currying & Partial Function Application
  - Types make separation of domain, solver easy

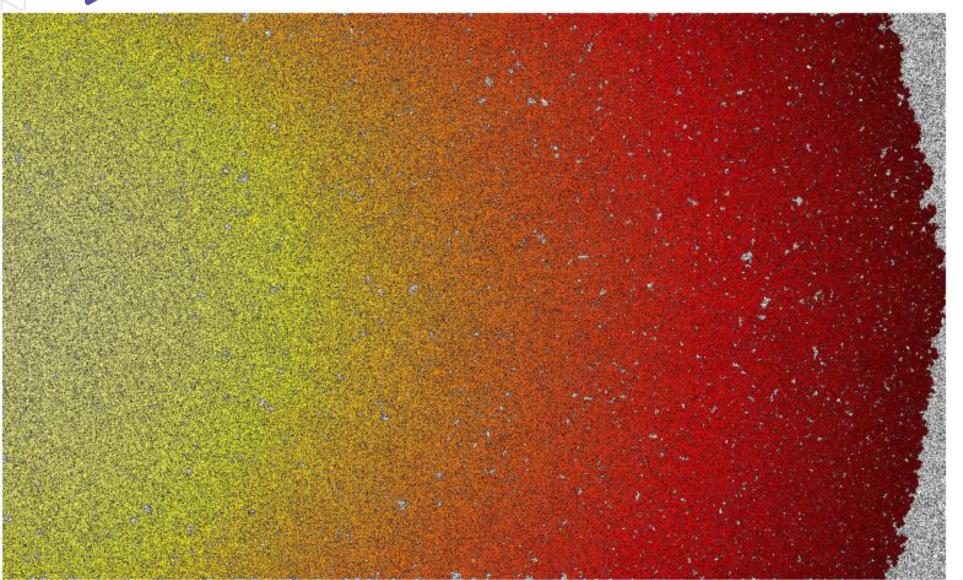
## Heuristic Search: Type Perspective

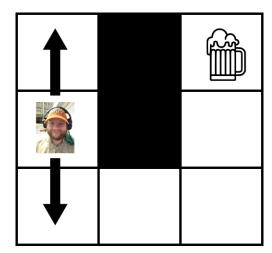
# > .

### Talk Outline

- Introduction
  - Grid Navigation Problem Setup
  - Example Domains Toys
  - Example Domains Industrial
- An Algorithm: A\*
- Fitting Domains To Interface
  - Grid Navigation
  - String Edit Distance
- Conclusion

# > Heuristic Search

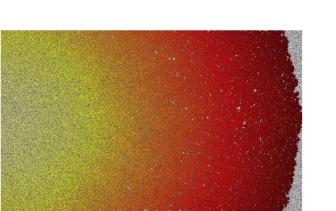








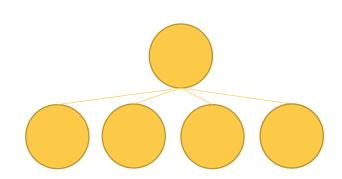
g(n)

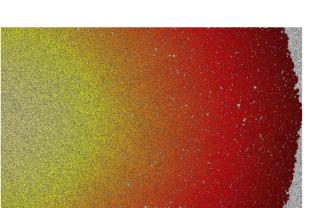






g(n)

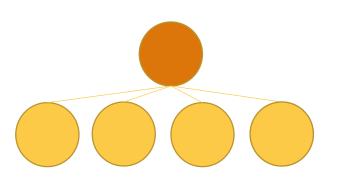


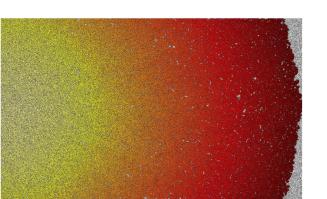








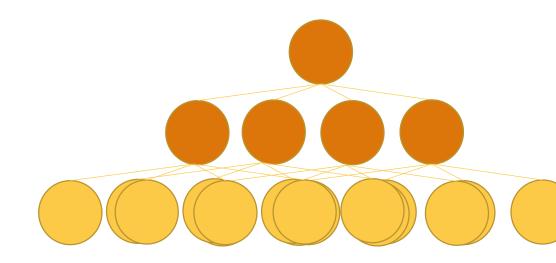




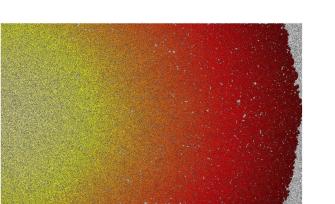




g(n)



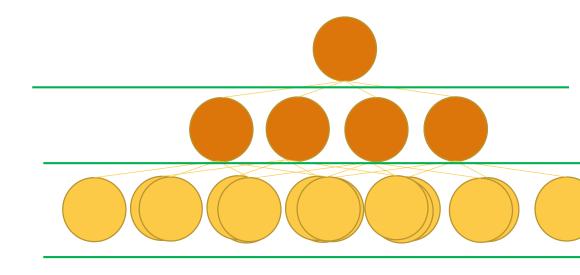
And so on and so forth...







g(n)



And so on and so forth...

